

Discrete Stochastic Programming by Infinitesimal Perturbation Analysis: the Case of Resource Allocation in Satellite Networks with Fading

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Abstract—This paper deals with a *NP-Hard* resource allocation problem for a satellite network. An approach based on the estimation of the gradient of a cost function, obtained through a “relaxed continuous extension” of the discrete constraint set, is proposed. Since neither closed forms of the performance measure, nor additional feedbacks on the statistical properties of the traffic sources are requested, the proposed approach reveals to be suitable for optimizing the resource allocation in real life case studies, where the application of specific *certainty equivalent* assumptions is impractical.

Index Terms—Satellite networks, resource allocation, fade countermeasures, gradient estimation.

I. INTRODUCTION

IN satellite networks, dynamically varying fading conditions over the channel can heavily affect the transmission quality, especially when working in *Ka band* (between 20 and 30 GHz), where the effect of rain fading is more significant. In the literature, it is possible to find optimal transmission policies, developed in the case of energy constraints for satellite network devices. The problem is usually analyzed and solved at the physical layer. Power allocation is performed, in order to obtain good reactions to variable fading conditions. Reference [1] shows a dynamic programming formulation of the problem that leads, for special cases, to a *closed-form* optimal policy. The latter finds a tradeoff between the minimization of the energy required to send a fixed amount of data and the maximization of the throughput over a fading channel. Other approaches need to locate the control system at both the physical and data link (or upper) layers, and provide adaptive bandwidth allocation strategies [2, 3]. In this case, the control systems are often based on closed-form expressions for the performance measure. For example, in [2], the *Tsybakov-Georganas* formula [4] for the cell loss probability in the presence of *self-similar* traffic is used. The main drawback of these approaches is due to the fact that conditions for the applicability of such closed-form expressions are difficult to implement in real-life contexts. In particular, the mapping between the current statistical behaviour of the system and the model parameters must be constantly updated, to maintain a good performance of the allocation algorithm. Many of

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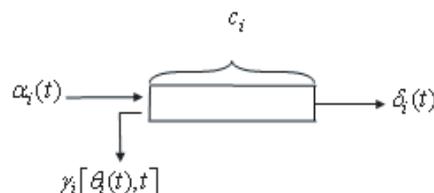


Fig. 1. The model of the i th satellite station.

the adopted techniques need also the application of *Dynamic Programming*, whose on-line implementation (needed for control adaptation) may become quite impractical, due to the well-known *curse of dimensionality* problem. In this work, we present a novel solution for the bandwidth allocation in a satellite environment, based on *Infinitesimal Perturbation Analysis* (IPA) [5] and on a *surrogate* relaxation of a discrete cost function [6,7]. IPA is suitable for the optimization problem under investigation, since it allows us to formulate a control scheme that is *light* (i.e., polynomial in the state space), *adaptive* (i.e., able to *learn* the statistical changes of the system) and *non-parametric* (i.e., able to optimize the system performance without any closed-form expression of the performance measure). The paper is organized as follows. In the next Section we formulate the discrete stochastic optimization problem for resource allocation in a satellite network with fading. Then, in Section III, we address our optimization approach and we test its performance by simulation analysis in Section IV. Finally, in Section V, we conclude the paper and discuss future work.

II. THE OPTIMIZATION PROBLEM

We consider a given number of earth stations, sharing a common satellite link. Each station i has a finite-capacity buffer of fixed size c_i and a single server with service rate $\theta_i(t)$, t being the (continuous) time variable. The stochastic processes associated with this model and useful for our optimization approach are: the *input flow* of the buffer $\alpha_i(t)$; the *output flow rate* $\delta_i(t)$ ($\delta_i(t) = \theta_i(t)$, if the buffer is not empty; $\delta_i(t) = \alpha_i(t)$, if $\alpha_i(t) < \theta_i(t)$, or $\delta_i(t) = \theta_i(t)$, if $\alpha_i(t) \geq \theta_i(t)$, otherwise); the *loss rate* process due to a full buffer $\gamma_i[\theta_i(t), t]$ (Fig. 1).

Even if the optimization technique we are going to investigate is not related to a specific behaviour of the traffic sources, we now introduce the traffic model employed in the following simulation results. In the last years, it has been demonstrated that packet-based traffic shows statistical characteristics very

close to “self-similar” processes [8]. We suppose that each input process $\alpha_i(t)$ is a self-similar stochastic process, generated by the superposition of some variable bit rate sources. For each station i , the statistical parameters that describe such process are: the *peak bit rate* of the on-off sources and the *burst arrival rate* $\lambda_{burst}^i = \frac{M_i}{\bar{\tau}_i + \bar{\sigma}_i}$ of the aggregated flow (namely, the average burst frequency “seen” by station i). $\bar{\tau}_i$ and $\bar{\sigma}_i$ are the mean time durations of the burst and of the silence periods, respectively; these periods are both Pareto distributed, in order to guarantee asymptotic self-similarity of the flows [8]. M_i is the number of on-off sources in the flow $\alpha_i(t)$. Our aim is to counteract variable fading levels and traffic load conditions. The stochastic processes involved with fading are assumed to be non-stationary. Therefore, the optimization algorithm has to dynamically adjust the bandwidth allocation, by adaptively following the current behaviour of the stochastic processes and by distributing the available channel capacity among the traffic stations. For each station i , the performance measure of interest is the *Loss Volume* L_i over the interval $[0, T]$: $L_i = \int_0^T \gamma_i[\theta_i(t), t] dt$. The satellite capacity serves N earth stations. The *overall Loss Volume* of the overall system is the sum of the contributions of each station, i.e.:

$$L = \left\{ \sum_{i=1}^N \int_0^T \gamma_i[\theta_i(t), t] dt \right\}. \quad (1)$$

If $\theta^d(t) = [\theta_1^d(t), \dots, \theta_N^d(t)]$ is the vector of the (discrete) service capacities allocated to each station at time t (from here on, we shall use the superscript d to stress that a quantity belongs to a “discrete set”), we have $\theta^d(t) \in \Theta^d$, with:

$$\Theta^d = \left\{ \theta^d(t) \in \mathbb{N}^N : \theta_i^d(t) = h_i(t) \cdot MAU \right\}, \quad (2)$$

and

$$\sum_{i=1}^N \theta_i^d(t) = K \quad (3)$$

where $h_i(t) \in \mathbb{N}$ and K is the total service capacity available for the satellite system. The allocated service rate for each station is a discrete number of “*Minimum Allocation Units*” (MAUs), i.e., the smallest portion of bandwidth that can be assigned to a station. The effect of fading is modeled as a reduction in the bandwidth actually “seen” by an earth station. The fading effect is represented by a variable $\phi_i(t)$, which shows how the bandwidth $\theta_i^d(t)$ is reduced. For each station i , at time t , the “real” $\theta_i(t)$ is $\theta_i(t) = \phi_i(t) \cdot \theta_i^d(t)$; $\phi_i(t) \in [0, 1]$; $i = 1, \dots, N$. The reduction of the bandwidth actually seen by the station is due to the increase in the redundancy required to maintain a fixed *Bit Error Rate* (BER). We suppose the presence of fading reactions located at the physical layer and managed by each earth station, in order to provide the desired BER, by adopting appropriate *Forward Error Correction* (FEC) codes (see, e.g., [2, 3] and references therein). The problem is to find the optimal bandwidth allocation function, $Opt \theta^d(t)$, such that the overall loss volume of the satellite system is minimized:

$$Opt \theta^d(t) = \arg \min_{\theta^d(t) \in \Theta^d, \forall t > 0} J[\theta^d(t)]; \quad (4)$$

$$J[\theta^d(t)] = E\{L(\theta^d(t))\}; \quad (5)$$

$$E\{L(\theta^d(t))\} = E \left\{ \sum_{i=1}^N \int_0^T \gamma_i[\phi_i(t) \cdot \theta_i^d(t), t] dt \right\}. \quad (6)$$

We denote by ω_i the generic sample path for station i , namely, a realization of the stochastic processes that characterize the temporal evolution of station i : $\alpha_i(t), \phi_i(t), i = 1, \dots, N$. The expectation is over all the feasible sample paths for each station i . The minimization in Eqs. (4)-(6) expresses a *NP-hard* discrete stochastic programming problem. Following the theoretical framework of [6,7], we address a new optimization algorithm. The idea is to exploit the current level of congestion in each station through IPA, during a time interval of observation (i.e., a decision epoch). Then, a new reallocation is performed at the end of each decision epoch through a gradient descent step.

III. THE ON-LINE SURROGATE OPTIMIZATION ALGORITHM

To perform the above-described procedure, we need the gradient of the cost function L with respect to the bandwidth allocation. With such a gradient, we can apply an on-line optimization descent, capable to optimally distribute the available channel capacity among the stations. From here on, we consider the service rate $\theta_i^d(t)$ as a piecewise constant function, driven by the reallocation mechanism. A new reallocation is performed at the end of a *decision epoch*. Within the κ -th decision epoch, the service rate is constant and it is denoted by $\theta^d(\kappa)$. As regards the gradient computation, the reader is referred to [5], where an IPA technique is investigated to compute derivative estimators for a traffic buffer as a function of the sample paths $\omega_i, i = 1, \dots, N$, of the system. No specific *certainty equivalent* assumptions are necessary for the system under investigation. The discrete constraint set Θ^d is “relaxed” into a continuous one Θ^c [6,7]:

$$\Theta^c \in \Theta^c, \Theta^c = \left\{ \theta_i^c \in \mathbb{R}^+; \sum_{i=1}^N \theta_i^c = K \right\} \quad (7)$$

Then, letting \hat{t} be the delay latency of the satellite system, each station i , for every $t = \kappa \hat{t}, \kappa = 1, 2, \dots$, must:

- 1) **observe** the buffer temporal evolution during the time interval $[(\kappa - 1)\hat{t}, \kappa\hat{t}]$, according to the current sample path ω_i and bandwidth allocation $\theta_i^d(\kappa - 1), \theta^d(\kappa - 1) \in \Theta^d$;
- 2) **compute** the gradient estimation $\frac{\partial L_i(\theta_i)}{\partial \theta_i} \Big|_{\theta_i(\kappa-1)}$, according to *Infinitesimal Perturbation Analysis* [5] (we now indicate explicitly the dependence of L_i on $\theta_i(\kappa)$ for the duration of the κ -th decision epoch);
- 3) **adjust** the value of its “bandwidth need”, by using the gradient method:

$$\widehat{\theta}_i^c(\kappa) = \widehat{\theta}_i^c(\kappa - 1) - \eta \cdot \phi_i \cdot \frac{\partial L_i(\theta_i)}{\partial \theta_i} \Big|_{\theta_i(\kappa-1)} \quad (8)$$

- 4) **communicate** the bandwidth need $\widehat{\theta}_i^c(\kappa)$ to the master station;
- 5) (for each station i that has the role of master station) by looking at the information received from the other stations (i.e., $\widehat{\theta}_j^c(\kappa - 1), j = 1, \dots, N, j \neq i$), and on the basis of the local bandwidth need $\widehat{\theta}_i^c(\kappa - 1)$, **obtain** $\theta^c(\kappa - 1)$ through $\theta^c(\kappa - 1) = \Pi[\theta^c(\kappa - 1)]$ ($\Pi[\theta] = \arg \min_{\theta' \in \Theta^c} \|\theta - \theta'\|$ is

a projection operator) and **convert** $\theta^c(\kappa - 1)$ to the nearest discrete feasible neighbor $\theta^d(\kappa)$, so that $\theta^d(\kappa) \in \Theta^d$; such conversion defines the bandwidth allocation for the satellite system in the time interval $[\kappa t, (\kappa + 1)t]$.

The derivative estimation is computed as (see [5]):

$$\left. \frac{\partial L_i(\theta_i)}{\partial \theta_i} \right|_{\theta_i(\kappa)} = \sum_{k=1}^{N_i^k} \left. \frac{\partial L_i^k(\theta_i)}{\partial \theta_i} \right|_{\theta_i(\kappa)} \quad (9)$$

$$\left. \frac{\partial L_i^k(\theta_i)}{\partial \theta_i} \right|_{\theta_i(\kappa)} = -[\nu_k^k(\theta_i(\kappa)) - \xi_k^k], \quad (10)$$

where $L_i^k(\theta_i)$ is the contribution to the loss volume of the k -th active period B_k^k of the buffer within the decision epoch $[\kappa, \kappa + 1]$, ξ_k^k is the starting point of B_k^k , ν_k^k is the instant of time when the last loss occurs during B_k^k , and N_i^k is the number of active periods at station i within the decision epoch $[\kappa, \kappa + 1]$.

The above procedure stems from the usage of $\left. \frac{\partial L_i(\theta_i)}{\partial \theta_i} \right|_{\theta_i(\kappa)}$, which is the measurable quantity derived from IPA. Actually, we are trying to approximate the gradient with respect to θ_i^c , which would require the application of *Perturbation Analysis* on the discrete parameter set, as in [6,7]. However, since we are considering a service rate, which can admit infinitesimal perturbations, whose effect is evaluated from the sample path, we have chosen to apply IPA and to perform what would be a descent step on the loss function without discretization. In step 4, as is shown in [6], the “nearest” feasible neighbor $\theta^d(\kappa + 1) \in \Theta^d$ of $\theta^c(\kappa + 1) \in \Theta^c$ can be determined, by using an $O(N + 1)$ algorithm (see, e.g., [7] for further details). The latter is based on the $N + 1$ discrete neighbours of $\theta^c(\kappa + 1) \in \Theta^c$, not necessarily all feasible, and on the selection of one of them, which satisfies the discrete constraint set Θ^d . The technical conditions necessary for the convergence of the employed stochastic gradient-based algorithm are related to the “unbiasedness” and “consistency” of the IPA gradient estimator within each decision epoch. Hence, we must assume that the stochastic processes are stationary and ergodic within each observation interval $[\kappa, \kappa + 1]$. Otherwise, the obtained sensitivity estimation would capture measures related to a non stationary stochastic environment, thus affecting the convergence of the gradient descent. The other conditions, related to the convergence of the algorithm, are: a decreasing behaviour of the gradient stepsize, the existence of a global (rather than local) optimum of the cost function (6) and the boundedness of the IPA estimator in the surrogate domain Θ^c . It is easily provable that such conditions hold true for the optimization framework under investigation. However, it is necessary to assume that the time-scale of the fading changes be lower than the time required for the convergence of the proposed control scheme. Otherwise, the resulting allocations would be only suboptimal. As we show in the following, this depends on the chosen gradient stepsize.

We now briefly summarize the allocation strategies employed in the following simulation results.

Just IPA: during the network’s lifetime, the IPA technique described previously is adopted, and derivative estimations are computed. After that, the aforementioned gradient algorithm is applied. No feedback about the state of the system, in terms of traffic load, is necessary to apply such optimization procedure.

In the simulation results, a bound on the optimal stepsize as a function of the channel capacity will be provided.

Proportional: if either a perfect off-line knowledge of the temporal behaviour of the stochastic processes of the system or a perfect and fast on-line feedback over the system’s state is available, it is possible to apply a good heuristic, in order to get a good approximation of the optimal solution with a minimal computational burden. In the hypothesis that the earth stations are affected by the same fading level, let:

$${}^t\theta_i^c = K \cdot \left(1 - \frac{b_i}{b^{Tot}}\right), \quad b^{Tot} = \sum_{h=1}^N b_h, \quad i = 1, \dots, N \quad (11)$$

be the component of the proportional bandwidth allocation, with respect to the traffic load changes, where (for $i = 1, \dots, N$):

$$b_i = \frac{\bar{\tau}_i + \bar{\sigma}_i}{\bar{\tau}_i} = \frac{\text{single source's Peak rate}}{\text{single source's Mean rate}} \quad (12)$$

is the *burstiness* of station i , a statistical property of the traffic load at station i . A good heuristic allocates more bandwidth to the stations where the sources have smaller levels of burstiness. A similar proportional allocation can be applied in the presence of fading level changes. In case that no traffic changes take place, let:

$${}^f\theta_i^c = K \cdot \left(1 - \frac{\phi_i}{\phi^{Tot}}\right), \quad \phi^{Tot} = \sum_{h=1}^N \phi_h, \quad i = 1, \dots, N \quad (13)$$

be the component of the proportional bandwidth allocation with respect to fading level changes only. Namely, the available bandwidth is proportionally distributed, in order to “protect” the stations that are receiving the highest channel degradations. Then, at each reallocation time κ , for each station i , the proportional strategy computes its current bandwidth need as:

$$\theta_i^c = K \cdot \frac{{}^t\theta_i^c(\kappa - 1) + {}^f\theta_i^c(\kappa - 1)}{\sum_{i=1}^N [{}^t\theta_i^c(\kappa - 1) + {}^f\theta_i^c(\kappa - 1)]}, \quad i = 1, \dots, N \quad (14)$$

As in the Just IPA procedure, each station i transmits its $\theta_i^c(\kappa - 1)$ to the other stations and, at time κ , $\theta_i^c(\kappa - 1)$ is converted to the discrete feasible neighbor $\theta_i^d \in \Theta^d$, with the algorithm employed at step 4 of the Just IPA technique. Note that the proportional strategy needs to maintain a perfect knowledge about both the burstiness of the traffic sources and the fading levels.

IV. SIMULATION RESULTS

To test the proposed strategies, we developed a C++ simulator for the queues of the satellite network. The performance of interest is the averaged sum of the *Loss Probabilities* (LPs) of each station at the end of the simulation. The following loss measures are averaged over 25 independent replications of the simulation scenario, to meet a confidence interval less than 1% of the estimated loss value for 95% of the cases. We first consider a scenario with 2 stations, in which traffic load changes are added to those of fading levels. Then, we further compare the proposed strategies, by scaling up the number of stations. As in [2], the traffic pattern follows a self-similar

TABLE I
 TRAFFIC CHANGES

Time Interval [min]	0.0-14.0	14.0-38.0	38.0-60.0
Station1, bursts/s	7.5	3.75	7.5
Station2, bursts/s	3.75	7.5	3.75

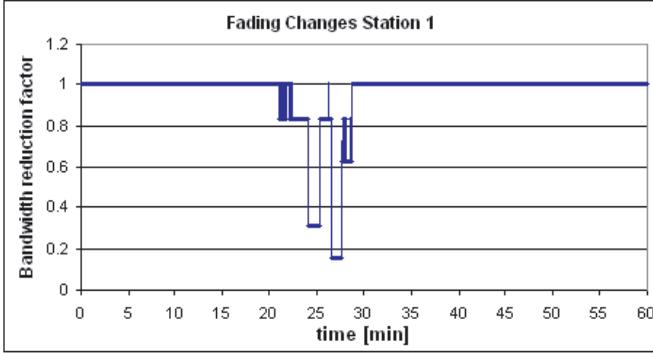


Fig. 2. Fading class changes: bandwidth reduction factor at Station 1.

behaviour, summarized in Section II. The employed fading processes come from [2], where real-life fading attenuation samples are taken from a data set, chosen from the results of experiments, in Ka band, carried out on the Olympus satellite by the CSTS (*Centro Studi sulle Telecomunicazioni Spaziali*) Institute, on behalf of the Italian Space Agency. The Carrier/Noise Power (C/N_0) ratio is monitored at each station and, on the basis of its values, different bit and coding rates are applied, in order to limit the BER below a chosen threshold of 10^{-7} . Six different fading classes are defined, corresponding to combinations of channel bit rate and coding rate. This gives rise to corresponding bandwidth reduction factors $\phi_i(t)$. With the data adopted in [2] we have (for $i = 1, 2$):

$$\phi_i(t) \in \{0.0, 0.15625, 0.3125, 0.625, 0.8333, 1.0\}. \quad (15)$$

The value $\phi_i(t) = 0$ corresponds to an outage condition; the value $\phi_i(t) = 1$ corresponds to clear sky. T is fixed to 1 hour and the fading effect does not affect Station 2, while it significantly deteriorates the quality of the channel for Station 1, as shown in Fig. 2. Only the IPA estimator is used to drive the gradient descent (8), disregarding the fading value (i.e., $\phi_i(t)$ is assumed to be 1 in (8)). The optimization procedure is capable to tune the bandwidth allocations in dependence of the working conditions, even without any knowledge on the redundancy applied at the physical level. This was also validated by other simulations, not reported in the paper. Traffic changes are defined in Table I and consider time-varying burst arrival rate conditions.

The highest burst arrival rate (7.5 bursts/s) is obtained with $M_i=15$, $\bar{\tau}_i=1.0$, $\bar{\sigma}_i=1.0$, $i = 1, 2$. The lowest one (3.75 bursts/s) with $M_i=15$, $\bar{\tau}_i=1.0$, $\bar{\sigma}_i=3.0$, $i = 1, 2$. Each station is provided with a buffer of 100 ATM cells, the total capacity of the system is fixed to 18.0 Mbps and each source is supposed to transmit at a peak bit rate of 1.0 Mbps. The MAU size is fixed at 100 kbps (correspondingly, $K=180$), the reallocation period is 1.0 second, and the gradient stepsize is $\eta_\kappa = [(5 \cdot 10^6) - \kappa \cdot (200 \cdot 10^3)]$. The fading peaks affect

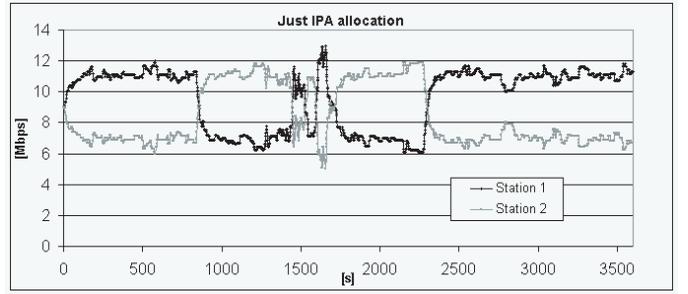


Fig. 3. Traffic load and real fading level changes. Just IPA allocation.

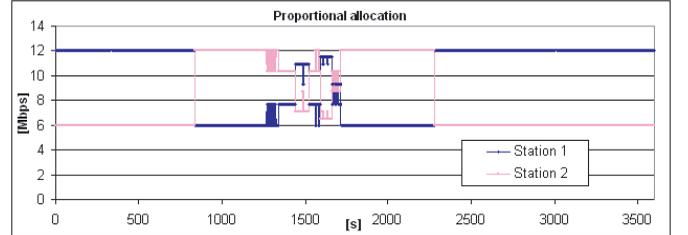


Fig. 4. Traffic load and real fading level changes. Proportional allocation.

Station 1 during its period of low traffic load; so, the proposed optimization strategies have to find out a good tradeoff in the bandwidth allocation with respect to both traffic load and fading changes.

As is shown in Fig. 3 and Fig. 4 (where the allocations corresponding to a specific realization are reported), both the Just IPA and the Proportional strategies are capable to change the resource allocation with respect to the variable system conditions. Just IPA guarantees a more balanced partition of resources with respect to the proportional assignment (the steady states in the bandwidth allocation for the station in high traffic load are around 11.0 Mbps for Just IPA and 12.0 Mbps for the Proportional technique). This slightly affects the system performance: the average LP assured by Just IPA is $1.15 \cdot 10^{-2}$ while the Proportional strategy guarantees an average LP of $1.38 \cdot 10^{-2}$. If a perfect knowledge about the statistical properties of the traffic sources is available (Proportional strategy), an on-line proportional assignment with respect to the traffic load and the fading levels could allow guaranteeing good performance. Clearly, a perfect knowledge about the statistical properties of the traffic sources is difficult to assure, in real-life contexts. For this reason, the adoption of adaptive resource allocation algorithms, like the Just IPA proposed here, reveals to be more suitable to optimize the performance.

Fig. 5 represents the percentage performance improvement obtained by the Just IPA technique over the Proportional strategy, by increasing both the number of stations and the channel capacity (from 80.0 Mbps with 3 stations to 270 Mbps with 10 stations), to assure a LP level around $1.0 \cdot 10^{-2}$. Variable traffic conditions are taken into account as in Table I, disregarding the fading effect. The best gradient stepsize has been found out by simulation inspection as function of the channel capacity. It is about 8% of the value of the available bandwidth for up to 5 stations and about 4%, when the number of stations is between 6 and 10. The obtained bandwidth allocations are similar to the ones depicted in Fig. 2 and Fig.

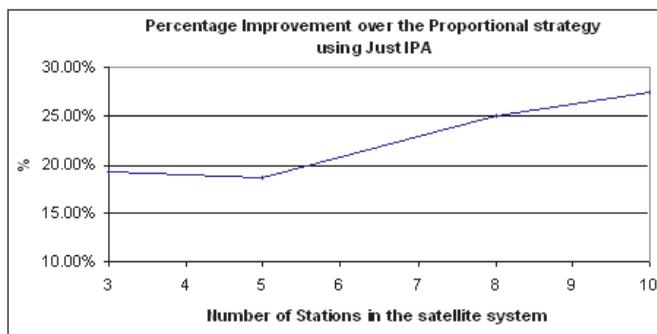


Fig. 5. Traffic changes and scaling up the number of stations in the system.

3. As shown in Fig. 4, even without any knowledge on both the fading and the traffic processes, Just IPA outperforms the Proportional allocation, when the number of stations increases. A comparison between Just IPA and a parameter adaptive *certainty equivalent* control (which makes use of a closed form expression of the LP performance measure) is reported in [9]. Even in this case, the application of IPA assures the best performance.

V. CONCLUSIONS AND FUTURE WORK

The application of the optimization methodology introduced in [6,7] has been proposed to react to fading and traffic load changes over a satellite network. It computes sensitivity estimations based only on the sample paths of the system and it reveals to be a promising technique, able to react in real

operating conditions. Future work can include the application of the employed sensitivity estimation technique, in order to solve resource allocation problems for other important QoS parameters, such as delay and delay jitter, and for different application scenarios.

REFERENCES

- [1] A. Fu, E. Modiano, and J. N. Tsitsiklis, "Optimal energy allocation and transmission control for communications satellites," *IEEE Trans. Networking*, vol. 11, no. 3, pp. 488-500, June 2003.
- [2] N. Celandroni, F. Davoli, and E. Ferro, "Static and dynamic resource allocation in a multiservice satellite network with fading," *Internat. J. Satell. Commun. Network.*, vol. 21, no. 4-5, pp. 469-487, July-Oct. 2003.
- [3] F. Alagöz *et al.*, "Fixed versus adaptive admission control in direct broadcast satellite networks With return channel systems," *IEEE J. Select. Areas Commun.*, vol. 22, no. 2, pp. 238-249, Feb. 2004.
- [4] B. Tsybakov and N. D. Georganas, "Self-similar traffic and upper bounds to buffer-overflow probability in an ATM queue," *Performance Evaluation*, vol. 32, no. 1, pp. 57-80, Feb. 1998.
- [5] Y. Wardi *et al.*, "Online IPA gradient estimators in stochastic continuous fluid models," *J. Optimization Theory and Applic.*, vol. 115, no. 2, pp. 369-405, Nov. 2002.
- [6] K. Gokbayrak and C. G. Cassandras, "Online surrogate problem methodology for discrete stochastic resource allocation problems," *J. Optimization Theory and Applic.*, vol. 108, no. 2, pp. 349-376, Feb. 2001.
- [7] K. Gokbayrak and C. G. Cassandras, "Generalized surrogate problem methodology for online stochastic discrete optimization," *J. Optimization Theory and Applic.*, vol. 114, no. 1, pp. 97-132, July 2002.
- [8] P. M. Krishna, M. V. Gadre, and B. D. Uday, *Multifractal Based Network Traffic Modelling*. Kluwer Academic Publishers, New York, NY, 2003.
- [9] F. Davoli, M. Marchese, and M. Mongelli "Optimal resource allocation in satellite networks: certainty equivalent approach versus sensitivity estimation algorithms," *Int'l. J. Commun. Syst.*, vol. 17, no. 10, pp. 935-962, Dec. 2004.