Capacity Bound of MOP-based Allocation with Packet Loss and Power Metrics in Satellite Communications Systems

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***Abstract*—The task of a capacity allocation policy is to determine the optimal quantity of capacity that has to be shared among the transmitting entities. In this work the allocation problem is modelled by the Multi Objective Programming (MOP) theory. In particular, an allocation criterion based on the L_p-*problem* is proposed to find out a capacity allocation, among Earth Stations, representative of a compromise if Packet Loss Probability and Transmitted Power are taken into account as performance metrics. The paper also discusses the existence of a capacity allocation, called Capacity Bound, on which the performance converges independently of the overall capacity available C_{TOT}. A performance analysis, carried out through simulations and under different satellite channel conditions, is finally proposed to investigate the allocation criterion performance and to show the Capacity Bound existence.**

***Index Terms*—Satellite Communications, Multi-Objective Programming, L_p-*problem* based Allocation, Capacity Bound, Performance Analysis.**

I. Introduction

THIS paper defines capacity allocation as a competitive problem where each entity (i.e., Earth Station) accessing the shared available capacity is "represented" by a group of functions called *objective functions*. Each of them needs to be optimised at cost of the others. These functions model physical quantities, such as Packet Loss Probability (PLP) and Transmitted Power (TP), as a function of the capacity allocated to the entity. If the functions are in contrast each other the allocation must represent necessarily a compromise. Modelling capacity allocation as described allows using Multi-Objective Programming (MOP) theory, which defines the multi-objective optimisation problem and the set of Pareto Optimal Points (POPs) as introduced in [1]. Each POP is often referred to a vector analogue for optimal solutions because the optimal solution for MOP is not formally defined. Optimal capacity allocations are chosen among POPs. Even if each POP is optimal from the Pareto viewpoint, to choose one solution is needed. A possibility, used in this paper, is the compromise solution [2], that selects a single POP minimising the distance, in the sense of the L_p-*problem* [3], with a reference goal point. In this paper, the solutions of the L_p-*problem* has been evaluated by applying several weights combinations as well as different norms to investigate several compromises among the adopted metrics: PLP and TP. Moreover, starting from the definition of L_p-*problem* and considering the specific analytical formulae of the *objective functions* adopted in this paper, explicitly reported in Section IV, the existence of a Capacity Bound has been formally and experimentally proven. In more detail, under the considered conditions, the L_p-*problem* provides the same solution (i.e., the allocation among entities does not change) even if the overall available capacity C_{TOT} tends to infinity. It means that the system performance does not change even if the resource availability, in terms of capacity expressed in [bps], significantly grows. Given a certain C_{TOT} and a given number of Earth Stations, the result allows considering the possibility to save capacity for other possible entities without performance detriment. The rest of the paper is organised as follows. The next section presents a brief survey of the state of the art about resources allocation for satellite and wireless communications systems. In Section III the MOP mathematical framework, used in this paper, is revised and the allocation criterion, modelled as the L_p-*problem*, are presented. Section IV describes the analytical formulae employed as performance metrics in this work: PLP and TP. Section V shows that the compromise solution is independent of the overall available capacity C_{TOT}, if it significantly grows, by demonstrating the existence of the Capacity Bound under the considered conditions in this paper. Section VI presents the simulation results that confirm the existence of the mentioned Capacity Bound analytically found. Finally the Conclusions are drawn.

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II. Brief Survey of the State of the Art

Since the last decade resources allocation for satellite and wireless communications systems is widely investigated. The most important resource considered by allocation algorithms is the capacity available, expressed in [bps], for data transmissions. The task of allocation algorithms concerned the maximisation of the capacity dedicated to each entity sharing the resource (i.e., the Earth Stations in this paper) aimed at improving the quality of communications. Recently, algorithms that consider capacity allocation and TP, simultaneously, have been introduced. In general, the algorithms available in the literature can be divided in two families. The first family concerns the capacity maximization (in [bps]), provided to the overall communications system (i.e., to all entities) by allocating to each entity a certain quantity of bandwidth, expressed in [Hz], and the power, in [W], useful to carry out the communication process. The total amount of bandwidth

and power assigned to the entities are constrained and the capacity is modelled as a function of bandwidth and power consumption according to the Hartley Shannon law. This approach can be found in [4] and [5]. The second family deals with the transmission power minimisation by allocating bandwidth (in [Hz]), and capacity (in [bps]). The TP by each entity, is analytically obtained by the Hartley Shannon Law. In such an approach the capacity is considered constrained over a given threshold to assure a certain level of communications quality. This approach can be found in [6] and in [7].
The main difference between the method in this paper, based on the MOP theory and similarly presented in [1], and the families above surveyed concerns the optimisation criterion used for the resource allocation. In more detail, our proposal tries to optimise the value of two (or more) metrics, simultaneously. The proposed MOP-based approach considers a single constrained control vector, the capacity allocated to each entity, and is explicitly aimed at optimising the Transmitted Power and a quality of service metric (the Packet Loss Probability in the case of this work), at the same time. On the contrary, the approaches found in the literature try to optimise a single metric (i.e., only one objective function), the capacity or the Transmitted Power, without explicit reference to quality of service parameters as done in this paper. Moreover, the proposed formulation allows showing the existence of a Capacity Bound that fixes the overall system performance and avoids possible capacity wasting.

III. The L_p-*problem* based Allocation

The model proposed in this paper is an extension of the proposals in [8], in [9] and [1] and is based on three main components: physical entities, virtual entities, and objective functions. [8] introduces the capacity allocation based on physical entities and objective functions; [9] opens the door to the concept of virtual entity by using more than one buffer for physical entity even if the term "virtual entity" is never mentioned. The main differences, introduced by this work, are the generalisation of the norm-based allocation method to find the POP compromise solution as suggested in [2], and, in particular, the proof of the existence of a Capacity Bound, under the considered conditions of this paper, detailed in Section V. As defined in [1], a physical entity is a device such as a satellite Earth Station. A virtual entity is a component within a physical device such as a single buffer-server. Each virtual entity is "represented" by a group of objective functions that model performance parameters such as, for instance, PLP and TP. Capacity allocations are performed by a centralised decision maker, which split C_{TOT} among all physical entities and assigned portions of capacity to virtual entities in dependence on the objective functions value.

In this section the capacity allocation problem is modelled as a MOP Problem previously formalised in [1] and here reported for the sake of completeness. The system is composed by Z physical entities; each physical entity is identified by $z \in [1, Z]$. Y_z is the number of virtual entities of the $z-th$ physical entity. Each virtual entity is identified by $y_z \in [1, Y_z]$. M_{y_z} is the number of objective functions for each virtual entity y_z. Each objective function, of a given $y_z - th$ virtual entity, is identified by the index $m \in [1, M_{y_z}]$. C_{y_z} is the capacity allocated to the virtual entity y of the physical entity z.

$$\mathbf{C} = (C_{1_1}, C_{2_1}, C_{3_1}, ..., C_{Y_1}, ..., C_{1_Z}, C_{2_Z}, C_{3_Z}, ..., C_{Y_Z}) \quad (1)$$

is the vector that contains the capacity allocated to each virtual entity. $C_z = \sum_{y=1}^{Y_z} C_{y_z}$ is the capacity allocated to physical entity z. $F_{m,y_z}(\mathbf{C})$ is the $m-th$ objective function, analytically defined in Section IV, of the $y-th$ virtual entity of the $z-th$ physical entity. The full set of objective functions is contained in the vector

$$\begin{aligned}\mathbf{F}(\mathbf{C}) = (&F_{1,1_1}(\mathbf{C}), ...\\ &F_{M_{1_1},1_1}(\mathbf{C}), ..., F_{1,Y_Z}(\mathbf{C}), ..., F_{M_{Y_Z},Y_Z}(\mathbf{C}))\end{aligned} \quad (2)$$

Given the definitions above and given C_{TOT} the available physical capacity, shared by all Z entities, the following constraint must hold:

$$\sum_{z=1}^{Z}\sum_{y=1}^{Y_z} C_{y_z} \leq C_{TOT} \quad (3)$$

Capacity allocation is defined as a MOP problem through (4), which must be solved under the constraint (3) that defines the feasibility region.

$$\begin{aligned}\mathbf{C}_{opt} = \Big(&C_{1_1,opt}, C_{2_1,opt}, .., C_{Y_1,opt}, ..,\\ &C_{1_Z,opt}, C_{2_Z,opt}, .., C_{Y_Z,opt}\Big) = \arg\min_{\mathbf{C}} \mathbf{F}(\mathbf{C});\\ &C_{y_z} \geq 0, \forall y_z \in [1, Y_z]. \forall z \in [1, Z]\end{aligned} \quad (4)$$

The set of solutions deriving from (4) is called POP set. In general, getting the overall POP set is not simple but the structure of the objective functions helps to take decision in some cases. For example, it is simple to prove that given the problem (4), subject to the constraint (3), if all objective functions are strongly decreasing [3], i.e. decreasing for all its variables and strictly decreasing for at least one function and one variable, then a solution $\mathbf{C}$ is a POP if and only if the solution is on the constraint boundary $\sum_{z=1}^{Z}\sum_{y=1}^{Y_z} C_{y_z} = C_{TOT}$. This is the case we have considered in [8] and [9]. It is also true that, given inequality constraint (3), if all objective functions are decreasing, all the points on the constraint boundary are POP solutions, but not all POP solutions necessarily belong to the constraint and also points for which $\sum_{z=1}^{Z}\sum_{y=1}^{Y_z} C_{y_z} < C_{TOT}$ can be POP solutions. The strongly decreasing assumption concerning the objective-function vector is quite typical because common performance functions applied in telecommunication networks such as Packet Loss Probability, Packet Delay and Packet Jitter are quantities that decrease their values when the allocated capacity value increases. This is not true if also other important metrics are used: power, but also processing and computation effort. It is simple to prove that, given problem (4) and constraint (3), if at least one function is strongly increasing, i.e. increasing for all its variables and strictly increasing for at least one variable, all the points inside the feasibility region as well on the constraint boundary may be

POP. The idea is to allocate capacity so that the value of each objective function is as close as possible to its ideal value. The set of ideal capacities (i.e. the ideal vector (5)) composed of the ideal decision variable vector elements $C_{y_z,id}^{F_{k,y_z}}$ for which F_{k,y_z} attains the optimum value, may be known having information about the features of the objective functions, as explained in the following. This definition of the ideal capacities set is not the only choice, e.g., if hard constraints on metrics were given, the ideal vector may contain the minimum capacity allocations so to assure these constraints.

$$\begin{aligned}\mathbf{C}_{id}^{F_{k,y_z}} &= \Big(C_{1_1,id}^{F_{k,y_z}}, C_{2_1,id}^{F_{k,y_z}}, ..., C_{Y_1,id}^{F_{k,y_z}}, ..., \\ &\quad C_{1_Z,id}^{F_{k,y_z}}, C_{2_Z,id}^{F_{k,y_z}}, ..., C_{Y_Z,id}^{F_{k,y_z}}\Big) \\ &\quad \forall k \in [1, M_{y_z}], \forall y_z \in [1, Y_z], \forall z \in [1, Z]\end{aligned} \tag{5}$$

Each element $C_{y_z,id}^{F_{k,y_z}}$ can assume a value between 0 and C_{TOT}, independently of any physical constraint and of the values of the other components of vector (5). It is called ideal (utopian) for this. For example, if a generic objective function is decreasing versus capacity, it is obvious that it is ideal allocating all the possible capacity C_{TOT}, while if it is increasing versus capacity, it is ideal allocating no capacity at all. The values of vector (5) are considered known in the remainder of the paper. Vector in (6) contains each objective function attaining its ideal value.

$$\begin{aligned}\mathbf{F}_{id} = \bigg(&F_{1,1_1,id}\Big(\mathbf{C}_{id}^{F_{1,1_1}}\Big), .., F_{k,y_z,id}\Big(\mathbf{C}_{id}^{F_{k,y_z}}\Big), .., \\ &F_{M_{Y_Z},Y_Z,id}\Big(\mathbf{C}_{id}^{F_{M_{Y_Z},Y_Z}}\Big)\bigg)\end{aligned} \tag{6}$$

The allocated optimal capacity based on the proposed criterion is reported in (7).

$$\begin{aligned}&\mathbf{C}_{all} = (C_{1_1,all}, C_{2_1,all}, ..., C_{Y_1,all}, ..., C_{1_Z,all}, C_{2_Z,all}, ..., \\ &C_{Y_Z,all}) = \arg\min_{\mathbf{C}\subset\mathbf{C}_{opt}} J_p(\mathbf{C})\end{aligned} \tag{7}$$

where

$$\begin{aligned}J_p(\mathbf{C}) = \Bigg(&\sum_{z=1}^{Z}\sum_{y=1}^{Y_z}\sum_{k=1}^{M_{y_z}} w_{k,y_z}\bigg|F_{k,y_z}(\mathbf{C}^{F_{k,y_z}}) - \\ &+ F_{k,y_z,id}\Big(\mathbf{C}_{id}^{F_{k,y_z}}\Big)\bigg|^p\Bigg)^{1/p}\end{aligned} \tag{8}$$

and $\sum_{k=1}^{M_{y_z}} w_{k,y_z} = 1$, $w_{k,y_z} \geq 0, \forall k \in [1, M_{y_z}]$, $\forall y_z \in [1, Y_z], \forall z \in [1, Z]$ so to assure the Pareto optimality of the solution as indicated in reference [3], page 98.

As extensively described in [2], the summation arguments in (8) can be considered in two ways: *i)* as transformations of the original objective functions; *ii)* as components of a distance function that minimizes the distance between the solution point and the ideal value, also called utopia point, in the criterion space. In practice, in this paper we minimise the distance (i.e., the norm) with respect to the utopia point, which gives origin to a POP solution [3] and is also known as Compromise Programming method. $J_p(\mathbf{C})$ is a function representing the generic norm, usually indicated with the symbol L_p [3], applied to calculate the distance from the ideal vector. In Section VI is reported a comparative performance analysis, carried out by varying norms and weights combinations, aimed at finding the better choice for the capacity allocation problem. The use of weights w_{k,y_z}, as well as different norms, allows allocating capacity to virtual entities by differentiating the importance of the performance metrics for different virtual entities up to neglecting one or more metrics, if necessary. This may be important to give more elasticity to capacity allocation also in dependence on the provided service (e.g., telephony, video-conferencing, audio/video streaming, web transactions) and on the provider/user requirements (e.g., capacity and energy costs, objective performance metrics versus P-QoS).

IV. The Objective Functions

In this paper each physical entity represents an Earth Station that transmits through a satellite channel. It is modelled as a single buffer (as a consequence, physical and virtual entities are not differentiated). Each considered entity is represented by two objective functions that are the Packet Loss Probability, shortly PLP, due to congestion ($F_{1,1_z} = P_{loss_z}(C_z)$) and the Transmitted Power, shortly TP, ($F_{2,1_z} = W_{tx_z}(C_z)$) and the constrain is defined by the amount of available capacity ($\sum_{z=1}^{Z}\sum_{y=1}^{Y_z} C_{y_z} \leq C_{TOT}$). The PLP model used in this paper deals with Transmission Control Protocol (TCP) based traffic and is analytically reported in (9) as defined in [10]:

$$P_{loss_z}(C_z) = \frac{k_z \cdot N_z^2}{(\frac{R_z \cdot C_z \cdot rtt_z}{l} + Q_z)^2} \tag{9}$$

In this paper, the values of the variable reported in (9), applied in the performance analysis section, and the related meanings are: k_z=128/81 is a constant depending on TCP parameters, N_z =10 is the number of active TCP connection for the *z-th* station, Q_z is the buffer size, equal to 10 packets, for the *z-th* station. rtt is the the round trip time, is equal to 512 [ms], l =1500 [byte] is the TCP packet size and R_z and C_z are the code rate and the capacity allocated to the *z-th* station, respectively. Channel conditions vary over the time and, in this paper, the experienced Carrier to Noise ratio $\left(\frac{C}{N}\right)_z$ for each station represents the satellite channel status. Each Earth Satellite station is supposed to apply different code rates in dependence on the channel status. Code rates are assigned as in Table I. This hypothesis allows considering packet losses due to congestion because channel errors are made negligible by applying encoding. In (10) we rewrite the equation (9) in a simpler form. It will be useful in Section V for an easier mathematical tractability:

$$P_{loss_z}(C_z) = \frac{A_z}{(D_z \cdot C_z + Q_z)^2} \tag{10}$$

TABLE I
APPLIED CODE RATES

$\left(\frac{C}{N}\right)_z$ [dB]	4.25-4.75	4.75-5.25	5.25-5.75	5.75-6.25	6.25-6.75
R_z	1/2	2/3	3/4	5/6	7/8

where $A_z(N_z) = k_z \cdot N_z^2$, $D_z(R_z) = \frac{Rc_z \cdot rtt}{l}$. The TP of the *z-th* station is reported in (11):

$$W_{tx_z}(\alpha_z, C_z) = (2^{\frac{C_z}{B}} - 1) \cdot \alpha_z \tag{11}$$

α_z, called link constant in this paper, takes into account the parameters related to the link budget. In more detail, it contains the transmission antenna gain G_{T_z} of the *z-th* station, the receiver antenna gain on the satellite G_R (common for each station) both equal to 10^4, the Boltzman costant *k* equal to $1.38 \cdot 10^{-23} J \cdot K^{-1}$, the noise temperature *T* set to 290 [K], the bandwidth of the satellite channel *B*=1 [MHz] and the Free Space Loss (*FSL*) set equal to 10^{19} as defined in [11]. In practice, the coefficient α_z is:

$$\alpha_z = \frac{k \cdot T \cdot B \cdot FSL}{G_{T_z} \cdot G_R} \tag{12}$$

The Transmitted Power function is obtained by combining two equations: $C_z = B \cdot log_2\left(1 + \left(\frac{C}{N}\right)_z\right)$ the Hartley-Shannon law, and $\left(\frac{C}{N}\right)_z = \frac{G_{T_z} \cdot G_R \cdot W_{tx_z}}{k \cdot T \cdot B \cdot FSL}$ that represents the carrier to noise ratio [11].

V. The Capacity Bound of the Compromise Programming

In this paper we consider Z physical entities, a single virtual entity for each physical entity ($Y_z = 1 \; \forall z \in [1, Z]$) and two objective functions for each virtual entity ($k = 2 \; \forall y_z \in [1, Y_z], \forall z \in [1, Z]$). Considering the two objective functions previously introduced, the vector $\mathbf{F}(\mathbf{C})$, defined in (2), can be written as

$$\mathbf{F}(\mathbf{C}) = \left(\frac{A_1}{D_1 C_1 + Q_1}, (2^{\frac{C_1}{B}} - 1)\alpha_1, ..., \frac{A_Z}{(D_Z C_Z + Q_Z)^2}, (2^{\frac{C_Z}{B}} - 1)\alpha_Z\right) \tag{13}$$

According to (6) the utopia points for the employed objective function are $F_{1,1_z,id} = \dfrac{A_z}{(D_z C_{TOT} + Q_z)^2}$ and $F_{2,1_z,id} = 0$. Consequently, the function $J_p(\mathbf{C})$, representing the L_p norm applied, that needs to be minimised to obtain the POP solution of the so called Compromise Programming problem is, in practice, a function of the vector $\mathbf{C}$ and of the totally available capacity C_{TOT}:

$$J_p(\mathbf{C}, C_{TOT}) = \left(\sum_{z=1}^{Z} w_{1,1_z}\Big(\frac{A_z}{(D_z C_z + Q_z)^2} - + \frac{A_z}{(D_z C_{TOT} + Q_z)^2}\Big)^p + w_{2,1_z}\Big((2^{C_z/B} - 1)\alpha_z\Big)^p\right)^{1/p} \tag{14}$$

The aim of this section is to show that, given fixed channel conditions, if the overall capacity available for the entire communications system significantly grows, the POP solution provided by solving (7), considering $J_p(\mathbf{C}, C_{TOT})$ as defined in (14), will not significantly change tending, in the sense of a horizontal asymptote, to a quantity called Capacity Bound C^{bound}. From a formal viewpoint,

$$C^{bound} = \sum_{z=1}^{Z} C_z^{bound}, C^{bound} < C_{TOT} \tag{15}$$

where C_z^{bound} is the portion of capacity allocated to the $z-th$ Earth Station when the overall allocation converges on the defined bound. The mentioned C^{bound} exist and is finished if $C_z^{bound} \; \forall z \in [1, Z]$ is a quantity independent of C_{TOT} when C_{TOT} tends to infinity. In practice, the following conditions must be satisfied:

Condition 1: The limits of the partial derivatives of the function $J_p(\mathbf{C}, C_{TOT})$ as C_{TOT} approaches to infinity are functions of the sole capacity vector $\mathbf{C}$:

$$\lim_{C_{TOT} \to \infty} \frac{\partial J_p(\mathbf{C}, C_{TOT})}{\partial C_z} = \partial J_{p,z}(\mathbf{C}), \forall z \in [1, Z] \tag{16}$$

Condition 2: C_z^{bound} must represent a coordinate of an equilibrium point:

$$C_z^{bound} = \{C_z \in [1, C_{TOT}) : \partial J_{p,z}(\mathbf{C}) = 0, \forall z \in [1, Z]\} \tag{17}$$

Condition 3: The Hessian matrix of the problem (7), $\mathbf{H}(\mathbf{C})$, must be positive-semidefinite:

$$\det\Big[\mathbf{H}(\mathbf{C})\Big] \geq 0, \forall C_z \in [0, C_{TOT}] \tag{18}$$

Obviously, the *Conditions 2* and *3* are related to the existence and uniqueness of the minimum of the functions $J_p(\mathbf{C}, C_{TOT})$ computed if $C_{TOT} \to \infty$.

In the specific case of this paper, considering the previously defined conditions, we firstly compute the gradient $\nabla J_p(\mathbf{C}, C_{TOT})$ and we set it equal to zero to obtain the mentioned $J_p(\mathbf{C}, C_{TOT})$ minimum. In (19) is reported the $z-th$ component of the gradient vector:

$$\frac{\partial J_p(\mathbf{C}, C_{TOT})}{\partial C_z} = \left(w_{1,1_z}\Big(\frac{A_z}{(D_z C_z + Q_z)^2} + - \frac{A_z}{(D_z C_{TOT} + Q_z)^2}\Big)^{p-1} \cdot \frac{-2A_z D_z}{(D_z C_z + Q_z)^3} + + w_{2,1_z}\Big(2^{C_z/B} - 1\Big)^{p-1} \cdot \frac{2^{C_z/B} ln(2) \alpha_z^p}{B}\right) \cdot \left(\sum_{z=1}^{Z} w_{1,1_z}\Big(\frac{A_z}{(D_z C_z + Q_z)^2} - \frac{A_z}{(D_z C_{TOT} + Q_z)^2}\Big)^p + + w_{2,1_z}\Big((2^{C_z/B} - 1)\alpha_z\Big)^p\right)^{\frac{1}{p} - 1} \tag{19}$$

As said about the $J_p(\mathbf{C}, C_{TOT})$ function, the $z-th$ component of the gradient is a function of $C_z, \forall z \in [1, Z]$, and C_{TOT}. In general, it means that the compromise solution is a function of C_{TOT}. If we consider a significant increasing of C_{TOT} (i.e., C_{TOT} tends to infinity) the contribute of the term $\dfrac{A_z}{(D_z C_{TOT} + Q_z)^2}$ decreases and tends to zero. Formally

$$\lim_{C_{TOT}\to\infty} \frac{\partial J_p(\mathbf{C}, C_{TOT})}{\partial C_z} = \Bigg(w_{1,1_z} \Big(\frac{A_z}{(D_z C_z + Q_z)^2} \Big)^{p-1} \cdot \\ \cdot \frac{-2 A_z D_z}{(D_z C_z + Q_z)^3} + w_{2,1_z} \Big(2^{C_z/B} - 1 \Big)^{p-1} \\ \cdot \frac{2^{C_z/B} ln(2) \alpha_z^p}{B} \Bigg) \cdot \Bigg(\sum_{z=1}^{Z} w_{1,1_z} \Big(\frac{A_z}{(D_z C_z + Q_z)^2} \Big)^p \\ + w_{2,1_z} \Big((2^{C_z/B} - 1) \alpha_z \Big)^p \Bigg)^{\dfrac{1}{p} - 1} \tag{20}$$

The expression defined in (20) shows that the solution of the equation $\nabla J_p(\mathbf{C}, C_{TOT}) = 0$ is independent of C_{TOT} (i.e., is constant with respect to C_{TOT}) if it significantly grows so satisfying *Condition 1*. Indeed, the expression found in (20) is $\partial J_{p,z}(\mathbf{C})$. In detail, the allocation that can be obtained by equation (20) depends only on the link constant, the protocol parameters and by the employed norm. Moreover, it is easy to proof that the *Conditions 2* and *3* can be easily satisfied. Obviously, the obtained C^{bound} represents a Capacity Bound, defined in (15), whose existence strictly depends on the conditions, in terms of objective functions, considered in this paper. From the practical viewpoint, a Service Provider may provide capacity allocations to the Z Earth Stations without employing the overall available capacity and may dedicate the rest of the capacity to other possible entities. It can be done without penalising the performance because the allocation represents a compromise (in the sense of [2]). On the other hand, the result allows designing the minimum amount of C_{TOT} needed to obtain a compromise solution among Z stations without capacity wasting.

VI. Performance Analysis

The scenario considered in this performance evaluation has been implemented through the *ns-2* simulator. It is composed by $Z = 2$ Earth Stations, that transmit TCP traffic over a common geostationary satellite channel. The overall duration of simulations is 300 [s]. The allocation is done each 5 [s] (i.e., allocation period), and the channel condition experienced by each station, expressed by $\left(\dfrac{C}{N}\right)_z$, is randomly varied (by following a uniformly distributed probability density function of the values reported in Table I) and kept constant in each allocation period. Each value of the capacity and of the performance metrics reported in the figures and tables below represents the average of the values obtained by a number of simulation runs aimed at guaranteeing a confidence interval of the 95%.The key concept of this paper is the Capacity Bound on which the compromise solution converges if the overall available capacity C_{TOT} significantly increases. As previously reported in Section V given the objective function that models the QoS, the Packet Loss Probability (PLP) $P_{loss_z}(C_z)$ (9), and the Transmitted Power (TP) $W_{tx_z}(\alpha_z, C_z)$, the compromise solution is not a function of the overall capacity C_{TOT} if it tends to infinity. In Fig. 1, the compromise solution is reported: it represents the capacity globally allocated to both the stations (i.e., the sum of the capacities allocated to the two stations) obtained by varying C_{TOT} in the interval $[1 - 10]$ [Mbps]. Four norms ($p = [1, 2, 3, 4]$) have been considered and both the metrics have been equally weighted ($w_{1,1_1} = w_{1,1_2} = 0.5$). The allocation of the overall available capacity has been reported as reference. The compromise solution stays on the

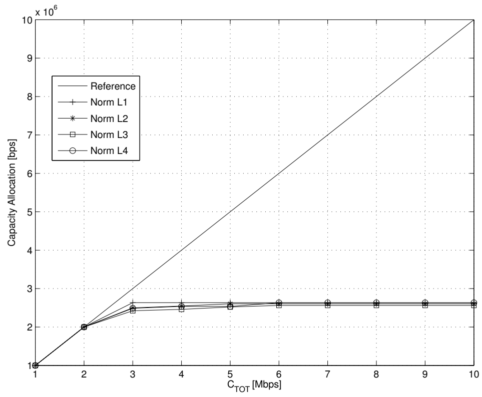

Fig. 1. Globally Allocated Capacity versus C_{TOT}.

constrain if $C_{TOT} \leq 2$ [Mbps]. If $C_{TOT} > 2$ [Mbps] the compromise solution is constant, around 2.6 [Mbps], for all the value of C_{TOT}. This is true for all the norms applied. It practically confirms the Capacity Bound whose existence has been discussed in Section V. The Capacity Bound has a significant impact on the performance. Fig. 2 shows that the PLP remains constant around 0.055. It is not far from the PLP level requested by many applications. In fact, as shown in Table III, the PLP level can be enhanced without impacting the TP significantly. The TP of a station is the

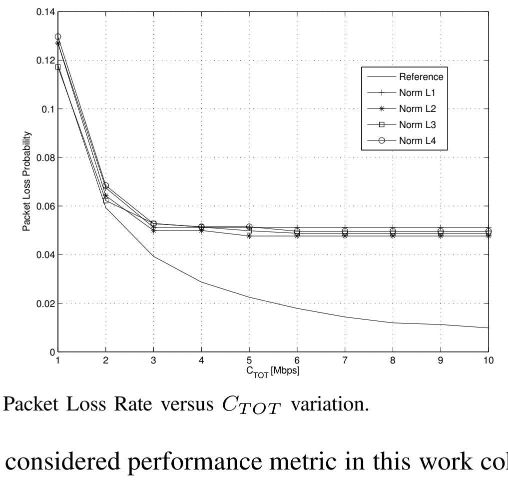

Fig. 2. Packet Loss Rate versus C_{TOT} variation.

second considered performance metric in this work coherently with Section IV. Its values are plotted in the Fig. 3. The TP is constantly lower than 0.1 [W] for each considered norm. This happens because the capacity allocated to each station, with the proposed method, is constant. If the allocations would follow the C_{TOT} behaviour the TP would grow exponentially. The proposed L_p*-problem* based Allocation criterion allows differentiating the solutions through weights applied to the

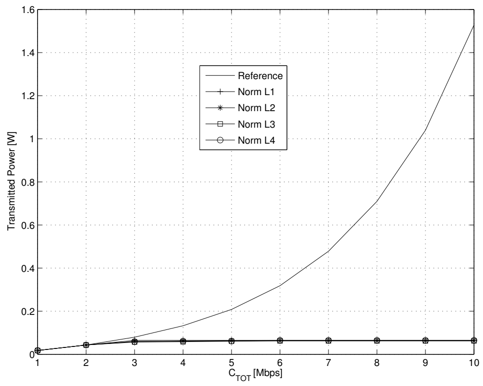

Fig. 3. Transmitted Power versus C_{TOT} variation.

objectives functions. Also the choice of a different norm, to compute the distance to the ideal point, may modify the POP position. In Table II the solutions, in terms of capacity globally allocated to the two stations, over norms and weights variations are reported. For all the applied norms, the most capacity saving weights combination, among the case considered in this work, is $w_{1,1_z}$ = 0.1, $w_{2,1_z}$ = 0.9, applied to both stations. This particular configuration assures also a reduction of power transmitted but also, at the same time, an increase of the packet loss rate, as reported in Table III and Table IV. Vice versa $w_{1,1_z}$ = 0.9, $w_{2,1_z}$ = 0.1 assures the minimum of the PLP but also the maximum TP, allocating the maximum of the capacity.

TABLE II
CAPACITY BOUND IN [BPS] VERSUS NORMS AND WEIGHTS VARIATIONS

$w_{1,1_z}$	0.9	0.75	0.5	0.25	0.1
$w_{2,1_z}$	0.1	0.25	0.5	0.75	0.9
Norm L_1	4805973	3698414	2763434	2035985	1328196
Norm L_2	3744290	3237478	2730666	2285021	1931127
Norm L_3	3368550	3012471	2686976	2453230	2202009
Norm L_4	3233109	2925090	2761250	2501290	2274099

TABLE III
PLP VERSUS NORMS AND WEIGHTS VARIATIONS

$w_{1,1_z}$	0.9	0.75	0.5	0.25	0.1
$w_{2,1_z}$	0.1	0.25	0.5	0.75	0.9
Norm L_1	0.02484	0.03425	0.051153	0.06913	0.10498
Norm L_2	0.03332	0.04113	0.047654	0.05864	0.07351
Norm L_3	0.03837	0.04373	0.048793	0.05839	0.06198
Norm L_4	0.04262	0.04507	0.049593	0.05329	0.06055

TABLE IV
TP IN [W] VERSUS NORMS AND WEIGHTS VARIATIONS

$w_{1,1_z}$	0.9	0.75	0.5	0.25	0.1
$w_{2,1_z}$	0.1	0.25	0.5	0.75	0.9
Norm L_1	0.17282	0.10504	0.06449	0.041283	0.02349
Norm L_2	0.10766	0.08353	0.06348	0.048509	0.038333
Norm L_3	0.08913	0.07446	0.06193	0.054092	0.045848
Norm L_4	0.08342	0.07080	0.06456	0.055427	0.048237

VII. CONCLUSIONS

The work proposes a capacity allocation criterion based on the L_p-*problem* representative of a compromise solution if Packet Loss Probability (PLP) and Transmitted Power (TP) are taken into account as performance metrics. Moreover, starting from the proposed L_p-*problem* allocation and considering PLP and TP defined as in Section IV, the paper highlights the existence of a Capacity Bound on which the allocations converge. The bound is independent of the overall capacity available C_{TOT}. The proposed performance analysis shows the performance of the proposed allocation approach and the Capacity Bound existence. It allows concluding that the proposed method enables a significant capacity and TP saving and, simultaneously, a limited worsening of the PLP. Practically, a Service Provider may provide capacity allocations to Z Earth Stations without employing its overall available capacity and may dedicate the rest of it to other possible entities without penalising the overall performance and avoiding satellite capacity wasting.

REFERENCES

[1] I. Bisio and M. Marchese, "Power saving bandwidth allocation over geo satellite networks," *Communications Letters, IEEE*, vol. 16, no. 5, pp. 596 –599, may 2012.

[2] R. T. Marler and J. S. Arora, "Survey of multi-objective optimization methods for engineering," *Structural and Multidisciplinary Optimization*, vol. 26, no. 6, pp. 369 –395, apr. 2004.

[3] K. M. Miettinen, *Nonlinear Multiobjective Optimization*. Boston, USA: Kluwer Academic Publisher, 1998.

[4] D. Chen and J. Laneman, "Joint power and bandwidth allocation in multihop wireless networks," in *Wireless Communications and Networking Conference, 2008. WCNC 2008. IEEE*, 31 2008-april 3 2008, pp. 990 –995.

[5] X. Gong, S. Vorobyov, and C. Tellambura, "Joint bandwidth and power allocation with admission control in wireless multi-user networks with and without relaying," *Signal Processing, IEEE Transactions on*, vol. 59, no. 4, pp. 1801 –1813, april 2011.

[6] X. Wang and G. Giannakis, "Power-efficient resource allocation for time-division multiple access over fading channels," *Information Theory, IEEE Transactions on*, vol. 54, no. 3, pp. 1225 –1240, march 2008.

[7] W. Ho and Y.-C. Liang, "Efficient resource allocation for power minimization in mimo-ofdm downlink," in *Vehicular Technology Conference, 2008. VTC 2008-Fall. IEEE 68th*, sept. 2008, pp. 1 –5.

[8] I. Bisio and M. Marchese, "Minimum distance bandwidth allocation over space communications," *Communications Letters, IEEE*, vol. 11, no. 1, pp. 19 –21, jan. 2007.

[9] ——, "Packet loss and delay combined optimization for satellite channel bandwidth allocation controls," in *Communications, 2008. ICC '08. IEEE International Conference on*, may 2008, pp. 1905 –1909.

[10] ——, "Analytical expression and performance evaluation of tcp packet loss probability over geostationary satellite," *Communications Letters, IEEE*, vol. 8, no. 4, pp. 232 – 234, april 2004.

[11] L. J. Ippolito, *Satellite Communications System Engineering*. Chichester, UK: John Wiley & Sons Ltd, 2008.